Adversarial Queueing Theory

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We study the behavior of communication networks in which packets are generated dynamically at the nodes and routed in discrete time steps across the edges. A crucial issue that arises in such a setting is that of *stability* — will the number of packets in the system remain bounded, as the system runs for an arbitrarily long period of time? The answer to this question typically depends on the *rate* at which packets are injected into the system, and on the contention resolution *protocol* that is used when more than one packet wants to cross a given edge in a single time step.

These issues have been investigated in a number of overlapping areas. Within the context of packet routing, there has been recent work focusing on the problem of stability in common inter-connection networks; typically, this work assumes that packets are generated according to independent Poisson or Bernoulli processes at the nodes, and they must be routed to random destinations [9, 10, 4, 8, 5]. In related work, Awerbuch and Leighton [1] presented a stable packet routing algorithm that could be used to provide a local–control approximation for the multicommodity flow problem. The problem of continuous packet injection and routing has also been a major topic of study within the field of queueing theory [6, 7]. Typical assumptions here are that packets are generated according to a Poisson process, and that the time to traverse an edge is an exponentially distributed random variable, rather than a fixed constant.

In joint work with Borodin, Raghavan, Sudan, and Williamson [3], we proposed a model of continuous packet injection in which probabilistic assumptions were replaced by worst–case inputs. Our goal was to determine whether it was feasible to prove stability results even when packets are injected by an *adversary*, rather than an oblivious randomized process. We termed this framework *adversarial queueing theory*, to reflect the fact that the emphasis is on *stability* — the central issue of queueing theory — with respect to an *adversarial* model of packet generation and path determination.

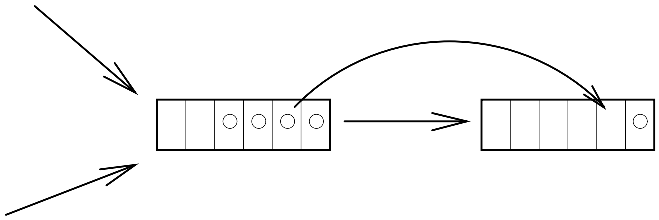

Figure 1: Advancing a packet

Thus, we consider the time evolution of a packet–routing network as a game between an *adversary* and a *protocol*. In each time step, the adversary *injects* a set of packets at some of the nodes; each packet has a specified sequence of edges that it must traverse, after which it will be *absorbed*. If more than one packet wishes to cross an edge e in the current time step, then the *protocol* chooses one of these packets to send across e; the remainder of these packets wait in a *queue* at the tail of e. This game then advances to the next time step. The protocol is said to be *stable* against the adversary if there is a constant C (possibly depending on the underlying network) so that there are never more than C unabsorbed packets in the system, regardless of how long the game is played.

A crucial parameter of the adversary is its *rate*. We define a single *request* by the adversary to be a set of packets requesting edge–disjoint paths; an adversary injects at *rate* r if for all t, no more than $\lceil rt \rceil$ requests are made in any interval of t steps. We raised a number of basic open questions in [3] concerning the relationship between rate and stability. In particular we asked,

(i) Does the n–node unidirectional ring have the property that every greedy protocol is stable against every adversary of rate less than 1?

*E-mail: kleinber@theory.lcs.mit.edu. The author is a student in the Theory of Computation Group, supported by an ONR Graduate Fellowship. This abstract is based on two papers. The first of these was done in part while visiting the IBM T.J. Watson Research Center, in collaboration with Allan Borodin, Prabhakar Raghavan, Madhu Sudan, and David Williamson; the second is joint work with Baruch Awerbuch, Antonio Fernández, Tom Leighton, and Zhiyong Liu.

(ii) Does *every* network have this property (namely that every greedy protocol is stable against every adversary of rate less than 1)?

(iii) Is *any* greedy protocol stable against every adversary of rate less than 1, for every network?

These questions highlight a very basic algorithmic question: when is a given contention resolution protocol stable in a given network, against a given adversary? More specifically, the questions are based on the following central definitions.

Definition. We say that a *graph* G is *universally stable* if every greedy protocol is stable against every adversary of rate less than 1 on G.

Definition. We say that *protocol* $\mathcal{P}$ is *universally stable* if it is stable against every adversary of rate less than 1, on every network.

We showed in [3] that directed acyclic graphs are universally stable; but for graphs with directed cycles, the following two extremes were both left open in [3] as possibilities: (a) every graph is universally stable; or (b) no graph containing a directed cycle is universally stable. If was also left open whether or not any or all greedy protocols are universally stable.

In a second paper, with Awerbuch, Fernández, Leighton, and Liu [2], we resolve these open questions of [3], and provide additional results on stability. In particular, we show that

(i) The n–node ring is universally stable, with maximum queue-size and maximum delay that is linear.

(ii) There exist commonly–used graphs and protocols that are not universally stable.

(iii) There exists a simple greedy protocol that is universally stable.

Our universal stability results (i) and (iii) hold for a much broader class of *bounded adversaries*, which we define as follows. The rate of an adversary in [2] is specified by a pair (w, r), where w is a natural number and $0 < r < 1$. The requirement on the adversary is the following: of the packets that the adversary injects in any interval of w steps, at most rw can have paths that contain any one edge. Such a model allows for adversarial injection patterns that are "bursty," since our rate restriction holds only in an amortized sense. In one time step, an adversary can inject a large number of packets that all request the same edge, provided simply that this does not result in more than rw packets requesting this edge over an interval of w steps. This is a much more robust and realistic assumption than the requirement that at most one set of edge–disjoint paths is injected in each time step.

Our example of instability in (ii) holds even for an adversary that injects at most one set of disjoint paths in each time step. The greedy protocol that turns out to be unstable in the construction of (ii) is simply the *Nearest–First* protocol, which always advances a packet whose distance to its destination is minimal. This is a protocol that has at times been proposed for routing in array–based parallel machines. Curiously, we show that many networks are unstable for the Nearest–First protocol, including arrays and hypercubes.

There is a protocol that is stable for all networks, however. In particular, we give a simple proof that every packet is delivered to its destination in a bounded amount of time, in any network, using the *Newest in System* protocol. (In this protocol, contention is resolved by giving precedence to the packet that was most recently injected into the network.)

We mentioned above that universal stability is a very basic algorithmic problem for graphs; hence we have the following natural question: Given a graph G, is it universally stable? Our results (i) and (ii) above show the non–triviality of universal stability as a property; and it is not initially clear that it should even be a *decidable* property, since it is asking whether *every* greedy protocol is stable against *every* bounded adversary on G. For undirected graphs with bi–directional edges, however, we show that universal stability is a decidable property; and in fact it can be decided in polynomial time. To prove this, we show that the set of universally stable graphs is closed under the taking of minors; polynomial–time decidability then follows from results of Robertson and Seymour [11].

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