Routing with Short Wires (Abstract)

Y. Lisa Zhang[1,3] D. Matthew Andrews[2,3]

We examine the problem of routing on two-dimensional grids using short wires. This has applications in several domains, e.g., integer multi-commodity flow, the global routing of gate-arrays in VLSI design and the routing of communication networks. Our approach is to relax an integer programming formulation of the problem to a linear program. We obtain a rounding of the fractional solution which is an improvement over that of Norton [2]. Related work has been done by Raghavan and Thompson [3].

Our routing problem is defined on an $n \times n$ grid of pins. Each instance specifies a collection of nets, where every net is specified by a pair of pins. The two pins of each net are to be joined using horizontal and vertical wires. In a *b-bend routing* the path for each net contains at most b turns. The *width* of a routing is defined to be the maximum number of nets routed along a particular edge in the grid. Given b, we would like to find a b-bend routing whose width is small. Karp *et al.* [1] have shown that even for a 1-bend routing it is NP-hard to minimize the width.

We consider a restriction of the problem, in which the Manhattan distance between the two pins of a net is bounded by a constant, k. We also restrict the path lengths to be at most k. (See Figure 1.) This model was first considered in the doctoral thesis of Norton [2]. She showed how to find a b-bend routing whose width, z, satisfies, $z \leq z_{\text{opt}} + O(b^2 \log^3 k)$, where z_{opt} is the optimal b-bend routing. The main result of this paper is an improvement of the error term to $O(b \log^2 k)$.

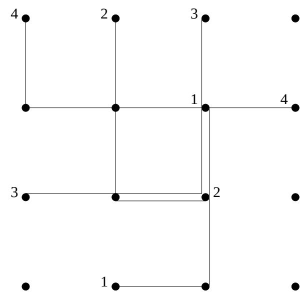

Figure 1: A 1-bend routing with $n = 4$, $k = 4$ and width $= 2$.

Our general approach is similar to that of Norton. For each net i we enumerate all the b-bend paths of length at most k between the two pins of the net. We use indicator variables x_{ij} to represent our routing, where $x_{ij} = 1$ if the jth path is used for net i, and $x_{ij} = 0$ otherwise. The problem can now be stated as an integer program:

$$\min z$$

subject to

$$\begin{array}{rcll} \sum_{i,j:e\in E(i,j)} x_{ij} & \leq & z & \forall \text{ grid edges } e \\ \sum_{\text{paths } j} x_{ij} & = & 1 & \forall \text{ nets } i \\ x_{ij} & \in & \{0,1\} & \forall i,j, \end{array}$$

where $E(i,j)$ is the set of grid edges used by path j of net i. This integer program can be

[1] Lisa Zhang is advised by Prof. Tom Leighton and supported by an NSF Graduate Fellowship.

[2] Matthew Andrews is advised by Prof. Michel Goemans and supported by NSF contract 9302476-CCR.

[3] Email: {ylz, andrews}@theory.lcs.mit.edu. Address: Rm 338, 545 Technology Square, Cambridge MA 02139.

relaxed to a linear program (LP) by replacing the constraints $x_{ij} \in \{0,1\}$ by $0 \leq x_{ij} \leq 1$. This LP can be solved in time polynomial in n, b and k. Our algorithm computes the optimum solution and then rounds it. Karp *et al.* [1] prove the following rounding theorem.

Theorem 1 *Let A be a real-valued $r \times s$ matrix, y and b be real valued vectors such that $Ay = b$, and let Δ be a positive real number such that in every column of A,*
(i) the sum of the positive elements is $\leq \Delta$,
(ii) the sum of the negative elements is $\geq -\Delta$.
Then we can compute an integral s-vector $\hat{y}$ such that
(i) $\hat{y}_i = \lfloor y_i \rfloor$ or $\hat{y}_i = \lceil y_i \rceil$ for $1 \leq i \leq s$,
(ii) $A\hat{y} = \hat{b}$, where $\hat{b}_i - b_i < \Delta$ for $1 \leq i \leq r$.

If we apply theorem 1 to the LP then $\Delta = k$. However, the rounded solution does not necessarily give a routing because we need $\sum_j \hat{x_{ij}} = 1\ \forall i$. In the special case of the matrix arising from the LP we can strengthen theorem 1 to the following.

Theorem 2 *Let A be the constraint matrix in the LP, x be the optimum solution, and $b = Ax$. Then we can compute $\hat{x}$ such that $\forall i$*
(i) $\hat{x_{ij}} \in \{0,1\}$,
(ii) $A\hat{x} = \hat{b}$, where $\hat{b_i} - b_i < \Delta$,
(iii) $\sum_j \hat{x_{ij}} = 1$.

It follows that a routing can be constructed whose width, z, satisfies $z \leq z_{opt} + O(k)$. The proof of theorem 2 contains the new ideas of this paper.

To improve the error bound, we use an idea from Norton [2] and introduce edge groups called *segments*. (See Figure 2.) Each row (column) of the grid has $\log k + 1$ levels of segments, in which the ith level has $(n-1)/2^k$ segments of length 2^i $(i = 0 \ldots \log k)$. A path *uses* a segment if it contains all the edges of the segment but does not contain all the edges in the next larger segment. Now each path between two pins of a net consists of $O(b \log k)$ segments, and each edge is contained in $O(\log k)$ segments. We replace the constraints

$$\sum_{i,j:e \in E(i,j)} x_{ij} \leq z\ \forall \text{ grid edges } e$$

by

$$\sum_{i,j:s \in S(i,j)} x_{ij} \leq z\ \forall \text{ segments } s,$$

where $S(i,j)$ is the set of segments used by path j of net i. The value of Δ for the new LP is $O(b \log k)$. This means that the number of nets using a segment is at most $O(b \log k)$ more than the optimum. Hence the width of the routing is at most $O(b \log^2 k)$ more than the optimum, since each edge is contained in $O(\log k)$ segments.

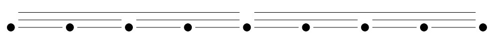

Figure 2: Segments for a row with $k = 4$.

This research is supervised by Tom Leighton and Michel Goemans. Serge Plotkin obtained the same error bound independently.

References

[1] R. M. Karp, F. T. Leighton, R. L. Rivest, C. D. Thompson, U. V. Vazirani, and V. V. Vazirani. Global wire routing in two-dimensional arrays. *Algorithmica*, 2:113–129, 1987.

[2] C. H. Norton. *Problems in Discrete Optimization.* PhD thesis, MIT, September 1993.

[3] P. Raghavan and C. D. Thompson. Randomized routing: A technique for provably good algorithms and algorithmic proofs. *Combinatorica*, 7(4):365–374, 1987.