Direct Computation of Reduced-Order Models for Circuit Simulation of 3-D Interconnect Structures*

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The dense three-dimensional packaging now commonly used in compact electronic systems may produce magnetic interactions which interfere with system performance. Such effects are difficult to simulate because they occur only as a result of an interaction between the field distribution in a complicated geometry of conductors, and the circuitry connected to those conductors. One approach to performing the coupled simulation is to use a 3-D field solver to compute the frequency-dependent impedance matrix associated with the geometry of conductors, and then use frequency-domain fitting and model-order reduction to construct a model suitable for circuit simulation.

In order to use frequency-domain fitting as described above, it is necessary to use the field solver to compute impedance matrices at dozens of frequency points, and this is computationally expensive. It is possible to derive a more efficient approach by exploiting the fact that 3-D field solvers typically use Krylov-subspace based iterative methods. These iterative methods can provide more than just a solution at a particular frequency, they can be used to directly construct reduced-order models.

A method of achieving this goal is to use the Arnoldi process directly on the state-space representation of the system of equations that describe the interconnect problem, represented by a triplet of matrices, $[\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}]$. The Arnoldi process generates a mutually orthogonal set of vectors which spans the space $\boldsymbol{A}^k\boldsymbol{B}, k = 0, \ldots, q-1$ [1]. The cost of each Arnoldi iteration is equivalent to that of a matrix vector product, which can be computed using a hierarchical multipole-algorithm as in FastHenry [2].

As the iterations proceed, the Arnoldi process constructs an orthogonal matrix $\boldsymbol{V}_q \in \mathbb{R}^{m\times q}$ and an upper Hessenberg matrix $\boldsymbol{H}_q \in \mathbb{R}^{q\times q}$. A q^{th} order model is then obtained as $\boldsymbol{G}_q^A(s) = \|\boldsymbol{B}\|\boldsymbol{C}^T\boldsymbol{V}_q\left(\boldsymbol{I} - s\boldsymbol{H}_q\right)^{-1}\boldsymbol{e}_1$ where s is complex frequency and $\boldsymbol{e}_1$ is the first column of the identity matrix. The eigendecomposition of $\boldsymbol{H}_q$ can be used to express $\boldsymbol{G}_q^A(s)$ as a rational function or by viewing $\boldsymbol{G}_q^A(s)$ as a differential equation, it can be stamped directly in to the circuit equations. We note that this approximation is *not* the traditional Padé approximation and that the computational cost of computing the Arnoldi model is smaller than that for an equivalent Padé approximation.

To compare the accuracy of the Arnoldi model to that of Padé, consider computing reduced-order models for the frequency-dependent admittance for a small set of package pins, as shown in Figure 1-a). Figure 1-b) shows the relative error of the 8^{th} order Padé and the 8^{th} and 16^{th} order Arnoldi approximations to the coupled admittance transfer function between pins 1 and 2. The figure indicates that the Arnoldi approximations are more accurate than the Padé approximation. We note that from the figure we can infer that the 8^{th} order

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Arnoldi approximation is more accurate than its equivalent Padé approximation of the same order, and it has half the computational cost, since only q GMRES iterations are performed.

Note also that if discrete frequency data were generated instead, and the approach described in [3] were used, generating 50 frequency points would require roughly 30 times the number of GMRES iterations needed by the direct state-space approach just described.

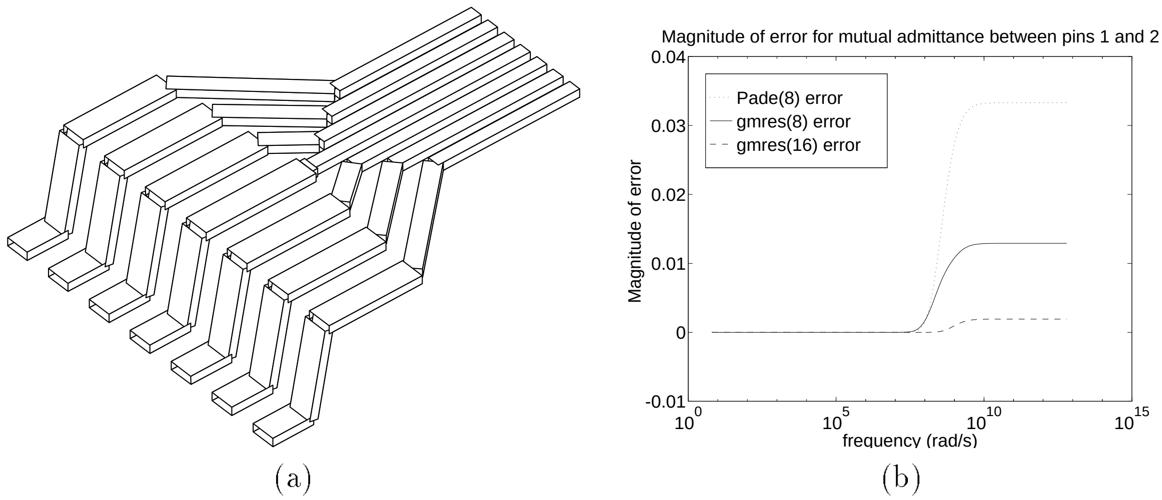

Figure 1: (a) Seven pins of a cerquad pin package. (b) Relative errors for the approximations $\boldsymbol{G}_8^P(s)$, $\boldsymbol{G}_8^G(s)$ and $\boldsymbol{G}_{16}^G(s)$ to the coupled admittance transfer function between pins 1 and 2

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